alex-levesque.com

PHIL 266 — Probability and Inductive Logic

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Based on lectures by J. Mozersky – Queen's University

Notes written by Alex Lévesque

These notes are my own interpretations of the course material and they are not endorsed by the lecturers.

Feel free to reach out if you point out any errors.

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1 Preface

Grading Scheme:

Textbook:

Comments:

Read this: https://www.maths.cam.ac.uk/undergrad/files/studyskills.pdf

2 Logic

2.1 Deductive Logic

Definition: In **deductive logic**, when you have true premises and a valid argument, the conclusion must be true too. Deductive logic does not take risks, i.e., there is no probability involved.

Inductive logic takes risks.

Definition: **Propositions** are statements that can be either true or false

Definition: An **argument** is a point or series of reasons presented to support a proposition which is the conclusion of the argument

Consider the arguments below:

Argument 1:
1. Either John is home or Sue is home.
2. John is not home.

So,

3. Sue is home

Argument 2:
1. Either Bill is happy or Kim is happy
2. Bill is not happy

So,

3. Kim is happy

These are different arguments but they have the same logical structure. Therefore, to get a sense of logical form, contrast it with grammatical form.

Definition: In logic, propositions are **true or false**. Arguments are **valid or invalid**. Strictly speaking, there is no such thing as a valid or invalid proposition, and no such thing as a true or false argument.

Definitions: A **logically necessary** argument cannot be false. A **logically impossible** argument cannot be true. A **logically contingent** is neither logically necessary nor impossible.

Definition: A **truth-preserving** argument is an argument where if all of its premises are true, then the conclusion must be true. A truth-preserving argument is valid.

Example: All humans are mortal. Socrates is a human. Therefore, Socrates is mortal. This is a truth-preserving argument. If the premises are true, the conclusion cannot be false.

2.1.1 Truth Functions

Truth functions

***Inclusive* 'or'**

P	Not-*P*
T	F
F	T

P	*Q*	*P* or *Q*
T	T	T
T	F	T
F	T	T
F	F	F

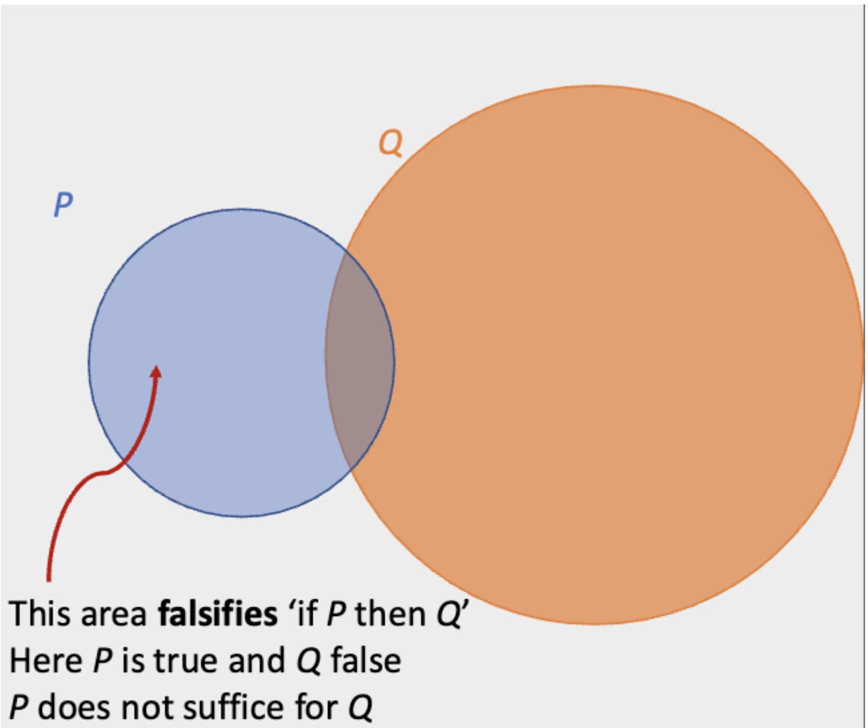
P	*Q*	*P* and *Q*
T	T	T
T	F	F
F	T	F
F	F	F

P	*Q*	If *P* then *Q*
T	T	T
T	F	F
F	T	T
F	F	T

P	*Q*	*P* if and only if *Q*
T	T	T
T	F	F
F	T	F
F	F	T

Considering *If* P *then* Q, we note that P is a subset of Q.

If, *if* P *then* Q is false, then there must be some region within P but outside Q.



2.1.2 Soundness

Definition: An argument is **sound** if and only if (1) it is valid and (2) all its premises are true. Sound arguments must have true conclusions.

Logic cannot tell us whether an argument is sound, but can tell us whether an argument is valid.

2.2 Inductive Logic

This is the introduction to uncertainty and risk in logic. We evaluate risky arguments using inductive logic.

Example: All math majors I have met went on to get good jobs. If I major in math, then I will get a good job. This outcome is actually uncertain and we must reason about risk to answer the question.

Definition: **Probability** is a fundamental tool for inductive logic. Say 95% of math majors get good jobs, so **probably** if I major in math, then I will get a good job.

3 Basic Concepts of Probability

It is important to understand what is meant by 'fair'. A biased coin is unfair. If is a pattern to an outcome, then this would be unfair. A fair set-up is (1) random, (2) has no memory of previous outcomes (it's irregular) and (3) is too complex for the next outcome to be predicted.

Definition: Trials on a chance set-up are **independent** if and only if the probabilities of the outcomes of the trial are not influenced by the outcomes of previous trials

Definition: A chance set-up is **fair** if and only if it is unbiased and outcomes are independent of each other

Gambler's Fallacy is the assumption that the set-up is fair, but also assumes that the set-up is not independent, so not fair.

Definition: Examples like tossed coins, dice, roulette wheels, etc. are simple and artificial **models** of real-life situations that can be much more complex.

Definition: An inference to a **plausible explanation** is a form of inductive reasoning in which you infer that a particular hypothesis best explains a set of observed facts.

Definition: An inference based on **testimony** is an inductive inference in which you accept a claim as likely true primarily because someone else asserts it.

3.1 Logical Notation and Language

In probability, we can discuss either **propositions** (statements that are t/f) or **events** (occurrences). These are equivalent in this context.

Concept	Logic Symbol	Set Theory	Meaning
Negation	$\sim A$	A'	Not A
Conjunction	$A\&B$	$A \cap B$	A and B (Both must occur)
Disjunction	$A \vee B$	$A \cup B$	A or B (At least one occurs)
Probability	$Pr(A)$	—	The probability that A occurs

3.2 Axioms of Probability

1. Range: For any outcome A, $0 \leq Pr(A) \leq 1$
2. Certainty: $Pr(\Omega) = 1$, where Ω is a guaranteed event
3. Addition Rule: If A and B are mutually exclusive (cannot happen at the same time), then $Pr(A \vee B) = Pr(A) + Pr(B)$
4. Multiplication Rule: If A and B are independent, then $Pr(A \wedge B) = Pr(A) \times Pr(B)$

3.3 Compound Events

Deck 1: 2 red cards, 3 black cards. H = die lands 5 or 6. R1 = draw red from first deck.

Deck 2: 4 red cards, 1 black card. L = die lands 1, 2, 3, or 4. R2 = draw red from second deck.

First, we throw the die. If 5-6, we pick from deck 1; If 1-4, we pick from deck 2.

What is the probability of drawing a red card?

$Pr(H\&R_1) = \frac{2}{6} \times \frac{2}{5} = \frac{2}{15}$

$Pr(L\&R_2) = \frac{4}{6} \times \frac{4}{5} = \frac{8}{15}$

$Pr(\text{Re}d) = Pr[(H\&R_1) \vee (L\&R_2)] = \frac{2}{15} + \frac{8}{15} = \frac{2}{3}$

4 Week 3